

Maximal atmospheric neutrino mixing in an $SU(5)$ model

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Received: 29 November 2002 /

Published online: 3 March 2003 – © Springer-Verlag / Società Italiana di Fisica 2003

Abstract. We show that maximal atmospheric and large solar neutrino mixing can be implemented in $SU(5)$ gauge theories, by making use of the $U(1)_F$ symmetry associated with a suitably defined family number F , together with a Z_2 symmetry which does not commute with F . $U(1)_F$ is softly broken by the mass terms of the right-handed neutrino singlets, which are responsible for the seesaw mechanism; in addition, $U(1)_F$ is also spontaneously broken at the electroweak scale. In our scenario, lepton mixing stems exclusively from the right-handed-neutrino Majorana mass matrix, whereas the CKM matrix originates solely in the up-type-quark sector. We show that, despite the non-supersymmetric character of our model, unification of the gauge couplings can be achieved at a scale $10^{16} \text{ GeV} < m_U < 10^{19} \text{ GeV}$; indeed, we have found a particular solution to this problem which yields results almost identical to the ones of the minimal supersymmetric standard model.

1 Introduction

The solar and atmospheric neutrino deficits – for a recent review see, e.g., [1] – are most naturally explained by neutrino oscillations [2], with matter effects playing a decisive role for solar neutrinos [3]. Whereas the favored solution of the solar neutrino problem, the large-mixing-angle MSW solution, displays a large but non-maximal mixing angle θ , the atmospheric neutrino problem with mixing angle θ_{atm} requires $\sin^2 2\theta_{\text{atm}} > 0.92$ at 90% CL [4]. It is not difficult to explain large (not necessarily maximal) atmospheric neutrino mixing – for reviews of mass-matrix textures for neutrino masses and lepton mixing see [5]. However, if the experimental lower bound on $\sin^2 2\theta_{\text{atm}}$ moves closer to 1, then the need for a symmetry to explain in a natural way $\theta_{\text{atm}} \simeq 45^\circ$ becomes acute. It is straightforward – see for instance [6–8] – that such a symmetry should be non-abelian. A few papers have attempted to explain nearly maximal atmospheric neutrino mixing in this way – for an incomplete list of references see [9–12]. Other approaches to this problem have also been suggested. For an interesting model with lopsided mass matrices see [13]. The renormalization-group evolution of the lepton-mixing angles from the grand unification scale m_U down to the electroweak scale m_Z (the mass of the Z boson) has been considered in many papers – see for instance [14–16] and the works cited therein. Interesting results can be obtained in this way [17], with part of the lepton-mixing problem tackled by the renormalization-group evolution while the

residual problem is left to be solved at the grand unification scale.

In the present paper we discuss the model for maximal atmospheric neutrino mixing introduced in [18], which belongs to the category of those using a non-abelian symmetry group (see Sect. 3 of [18]). For simplicity, let us call that model the maximal atmospheric mixing model (MAMM). Our aim in this paper is to show that the MAMM, which is a simple extension of the standard model (SM), can be embedded in a grand unified theory (GUT) based on the gauge group $SU(5)$ [19] (for a textbook see, e.g., [20]; for recent papers on the minimal supersymmetrized $SU(5)$ GUT see [21]). This “prototype GUT” can be considered as a testing ground for ideas on neutrino masses and mixing – for a recent paper see, e.g., [22].

First we summarize the MAMM. It concerns only the lepton sector of the SM, with its gauge group which we abbreviate as

$$G_{\text{SM}} = SU(3) \times SU(2) \times U(1). \quad (1)$$

There are the three well-known lepton families and, in addition, three right-handed neutrino singlets ν_R with a Majorana mass term

$$\mathcal{L}_M = \frac{1}{2} \nu_R^T C^{-1} M_R^* \nu_R - \frac{1}{2} \bar{\nu}_R M_R C \bar{\nu}_R^T, \quad (2)$$

where C is the charge-conjugation matrix and M_R is symmetric. We implement the seesaw mechanism [23] by assuming that $M_R^\dagger M_R$ is non-singular and that all its eigenvalues are of order m_R^2 , with $m_R \gg m_Z$. This leads to the effective Majorana mass matrix

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$$\mathcal{M}_\nu = -M_D^T M_R^{-1} M_D \quad (3)$$

for the light neutrinos. In (3), M_D is the Dirac mass matrix of the neutrinos. Allowing for an arbitrary number n_H of Higgs doublets, we avoid flavor-changing neutral Yukawa interactions by requiring that all the Yukawa-coupling matrices be diagonal – hence, M_D too is diagonal. This procedure is “natural,” since it amounts to conservation of the three lepton numbers L_e , L_μ , and L_τ in the Lagrangian. The only exception to this conservation is the Majorana mass term in (2), where the lepton numbers are allowed to be broken *softly*. Despite the soft breaking of the lepton numbers L_α ($\alpha = e, \mu, \tau$) at the high scale m_R , the resulting theory is well-behaved with respect to flavor-changing interactions and, moreover, it exhibits an interesting non-decoupling of the neutral scalar interactions for $m_R \rightarrow \infty$ when $n_H \geq 2$ [24]. In this framework, maximal atmospheric neutrino mixing is implemented by the symmetry

$$Z_2 : \quad \nu_{\mu R} \leftrightarrow \nu_{\tau R}, \quad D_\mu \leftrightarrow D_\tau, \quad \mu_R \leftrightarrow \tau_R, \quad (4)$$

where D_α denotes the left-handed lepton doublets. Because of Z_2 we have

$$(M_R)_{e\mu} = (M_R)_{e\tau}, \quad (M_R)_{\mu\mu} = (M_R)_{\tau\tau}, \quad (5)$$

and

$$M_D = \text{diag}(a, b, b). \quad (6)$$

As a consequence, the light-neutrino Majorana mass matrix of (3) has the same structure as M_R :

$$\mathcal{M}_\nu = \begin{pmatrix} x & y & y \\ y & z & w \\ y & w & z \end{pmatrix}. \quad (7)$$

Maximal atmospheric neutrino mixing and $U_{e3} = 0$ immediately follow from this structure of \mathcal{M}_ν . We stress that this structure results from a symmetry and that the MAMM, therefore, is really a model in the technical sense, not just a texture¹. Using an adequate phase convention and dropping possible Majorana phases, from the \mathcal{M}_ν of (7) we obtain the lepton-mixing matrix

$$U = \begin{pmatrix} \cos \theta & \sin \theta & 0 \\ \sin \theta/\sqrt{2} - \cos \theta/\sqrt{2} & -1/\sqrt{2} \\ \sin \theta/\sqrt{2} - \cos \theta/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}. \quad (8)$$

Since $m_\mu \neq m_\tau$, the Z_2 symmetry of (4) must be broken spontaneously by the vacuum expectation value (VEV) of some Higgs doublet transforming non-trivially under Z_2 . To avoid destruction of the form in (7) of the light-neutrino mass matrix, such a Higgs doublet must not contribute to M_D , but only to the mass matrix of the charged leptons. In [18] this problem was solved by having altogether three Higgs doublets and an additional Z'_2 symmetry; since that solution cannot be directly transferred to an $SU(5)$ model, we shall not discuss it in detail here.

¹ The charged-lepton mass matrix remains diagonal because of the assumed conservation, in all dimension-4 couplings, of the three lepton numbers L_α

The Z_2 of (4) does not commute with the $U(1)$ associated with the lepton numbers L_μ and L_τ . It is easy to see that we have in the MAMM the horizontal non-abelian symmetry group

$$U(1)_{L_e} \times U(1)_{(L_\mu+L_\tau)/2} \times O(2)_{(L_\mu-L_\tau)/2}. \quad (9)$$

We have indicated the lepton-number combinations associated with the $U(1)$ groups; the $O(2)$ is generated by

$$\begin{pmatrix} e^{i\alpha(L_\mu-L_\tau)/2} & 0 \\ 0 & e^{-i\alpha(L_\mu-L_\tau)/2} \end{pmatrix} \quad (\alpha \in \mathbb{R})$$

and

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad (10)$$

corresponding to $U(1)_{(L_\mu-L_\tau)/2}$ and Z_2 , respectively.

In this paper we shall discuss how the main features of the MAMM, namely the groups $U(1)_{L_\alpha}$ softly broken by \mathcal{L}_M and the symmetry Z_2 , can be embedded in an $SU(5)$ GUT. This will require a discussion of Z_2 breaking within $SU(5)$ with the purpose of allowing for a non-trivial CKM matrix. These subjects will be dealt with in Sect. 2. Since we shall end up with a proliferation of scalar multiplets, and since our model is in principle non-supersymmetric, gauge-coupling unification in the MAMM $SU(5)$ embedding is a non-trivial undertaking. Possible solutions to this problem will be studied in Sect. 3. Finally, our conclusions are presented in Sect. 4. The two appendices contain $SU(5)$ technicalities; the Yukawa couplings of the scalar $SU(5)$ -plets which we need and the charged-fermion mass matrices are given in Appendix A; in Appendix B we collect the branching rules with respect to G_{SM} of some irreducible representations (irreps) of $SU(5)$.

2 Maximal atmospheric neutrino mixing in $SU(5)$

2.1 $SU(5)$ preliminaries

The chiral fermion fields of one SM family are accommodated in $SU(5)$ irreps in the following way [19, 20]: $\psi_R \sim \underline{5}$ and $\chi_L \sim \underline{10}$, where the 10-plet is obtained as the anti-symmetric part of $\underline{5} \otimes \underline{5}$. The $\underline{5}$, which is the defining representation of $SU(5)$, has the generator of electric charge

$$Q_{\underline{5}} = \text{diag}(-1/3, -1/3, -1/3, +1, 0). \quad (11)$$

The fermion multiplets, in terms of chiral SM fields, are given by

$$\psi_R = \begin{pmatrix} d_R^1 \\ d_R^2 \\ d_R^3 \\ C\bar{\ell}_L^T \\ -C\bar{\nu}_L^T \end{pmatrix}, \quad (12)$$

$$\chi_L = \begin{pmatrix} 0 & C\bar{u}_R^3{}^T & -C\bar{u}_R^2{}^T & -u_L^1 & -d_L^1 \\ -C\bar{u}_R^3{}^T & 0 & C\bar{u}_R^1{}^T & -u_L^2 & -d_L^2 \\ C\bar{u}_R^2{}^T & -C\bar{u}_R^1{}^T & 0 & -u_L^3 & -d_L^3 \\ u_L^1 & u_L^2 & u_L^3 & 0 & -C\bar{\ell}_R^T \\ d_L^1 & d_L^2 & d_L^3 & C\bar{\ell}_R^T & 0 \end{pmatrix},$$

where the upper indices 1, 2, 3 are colour- $SU(3)$ indices. Note that $\chi_L^{ij} = -\chi_L^{ji}$.

The scalar $SU(5)$ multiplets which may couple to fermionic bilinears are determined by the following tensor products, allowed by the chiral structure of ψ_R and χ_L [20, 25]:

$$\underline{5} \otimes \underline{5} = \underline{15} \oplus \underline{10}, \quad (13)$$

$$\underline{5}^* \otimes \underline{10} = \underline{5} \oplus \underline{45}^*, \quad (14)$$

$$\underline{10} \otimes \underline{10} = \underline{5}^* \oplus \underline{50} \oplus \underline{45}. \quad (15)$$

The only scalar multiplets needed for our Yukawa couplings transform according to the irreps $\underline{5}$ and $\underline{45}$, or their complex conjugates [20]; see Appendix A for the construction of their Yukawa-coupling Lagrangians. In the following, the scalar 45-plets will be distinguished from the scalar 5-plets by a tilde.

All fermionic multiplets appear threefold, thus with family indices $a = 1, 2, 3$ they are denoted ψ_{Ra} , χ_{La} , and ν_{Ra} . The right-handed neutrinos are $SU(5)$ singlets: $\nu_{Ra} \sim \underline{1}$.

2.2 The family number

Implementing the idea of [18], we want to have M_D (the neutrino Dirac mass matrix) and M_ℓ (the charged-lepton mass matrix) simultaneously diagonal. This must be enforced by means of some symmetry. If M_ℓ is diagonal because of a symmetry, then we see from (A12) that the Yukawa-coupling matrices Y_d and \tilde{Y}_d must be diagonal; but then, from (A11), M_d turns out to be diagonal too. This means that quark mixing must stem exclusively from M_u , in the same way that lepton mixing originates exclusively from M_R .

Let us assume that, in analogy to the MAMM, there is only one 5-plet H_R coupling to the ν_R . Then we have the following terms in the Lagrangian (see, for instance, [20, 26]):

$$\bar{\nu}_{Ra} C (\bar{\psi}_{Rbi})^T H_R^i (Y_\nu)_{ab} - \frac{1}{2} \bar{\nu}_R M_R C \bar{\nu}_R^T + \text{H.c.} \quad (16)$$

The seesaw mechanism is operative and the light-neutrino mass matrix is given by (3), with

$$M_D = v_R Y_\nu / \sqrt{2}, \quad (17)$$

where $v_R / (2^{1/2})$ is the VEV of H_R . We introduce the family-number symmetry

$$F = \text{diag}(0, +1, -1), \quad (18)$$

applying both to the ψ_{Ra} and to the χ_{La} . In the ν_{Ra} sector one has $F = \text{diag}(0, -1, +1)$ instead. The scalar multiplet H_R coupling to $\bar{\nu}_R C \bar{\psi}_R^T$ (see (16)) and all the scalar multiplets coupling to $\bar{\psi}_R \chi_L$ (see Appendix A) are assumed to have $F = 0$. The family numbers of the scalar multiplets coupling to $\chi_L^T C^{-1} \chi_L$ (see Appendix A) will be discussed later. The symmetry group $U(1)_F$ defined here overtakes the role of the three groups $U(1)_{L_\alpha}$ of the MAMM. As a consequence of the symmetry $U(1)_F$, the Yukawa-coupling matrices Y_ν , Y_d , and \tilde{Y}_d are all forced to be diagonal, as we wanted; $U(1)_F$ is softly broken by the Majorana mass terms of the right-handed neutrinos, i.e. by M_R .

2.3 Maximal atmospheric neutrino mixing

In analogy to (4), we next introduce an interchange symmetry between the second and third families:

$$Z_2: \quad \psi_{R2} \leftrightarrow \psi_{R3}, \quad \chi_{L2} \leftrightarrow \chi_{L3}, \quad \nu_{R2} \leftrightarrow \nu_{R3}. \quad (19)$$

This forces $(Y_\nu)_{22} = (Y_\nu)_{33}$ and therefore leads to M_D of the form in (6). The matrix M_R moreover satisfies $(M_R)_{12} = (M_R)_{13}$ and $(M_R)_{22} = (M_R)_{33}$, just as in (5). Therefore \mathcal{M}_ν is as in (7) and we have maximal atmospheric neutrino mixing implemented. Note that $U(1)_F$ together with the Z_2 of (19) generate a symmetry group $O(2)$.

2.4 The down-type-quark and charged-lepton masses

We must check whether the introduction of $U(1)_F$ and of Z_2 is not incompatible with the freedom necessary to accommodate all the charged-fermion masses and CKM mixing angles. The CKM matrix is not the unit matrix, therefore the up-type-quark mass matrix M_u cannot be diagonal, contrary to what happens with the down-type-quark mass matrix M_d ; this implies that we must allow for non-diagonal Yukawa-coupling matrices for fermionic bilinears of the type $\chi_L^T C^{-1} \chi_L$. In order to obtain this, it is useful to separate the scalar multiplets coupling to $\bar{\psi}_R \chi_L$ from those coupling to $\chi_L^T C^{-1} \chi_L$. Furthermore, as we shall see below, in order to reproduce the down-type-quark masses while avoiding the destruction of the form of M_D in (6), one also needs to ensure that H_R is the only scalar multiplet coupling to the ν_{Ra} . In order to reproduce the down-type-quark masses and the charged-lepton masses we need two 5-plets H and H' together with one 45-plet \tilde{H} . We introduce the symmetries

$$Z'_2: \quad \nu_R \rightarrow -\nu_R, \quad H_R \rightarrow -H_R, \quad (20)$$

$$Z''_2: \quad \chi_L \rightarrow -\chi_L, \quad H \rightarrow -H, \quad H' \rightarrow -H', \\ \tilde{H} \rightarrow -\tilde{H}, \quad (21)$$

which allow for couplings of H_R only to $\bar{\nu}_R C \bar{\psi}_R^T$ (see (16)), while H , H' , and \tilde{H} couple only to $\bar{\psi}_R \chi_L$. All the scalar multiplets coupling to $\chi_L^T C^{-1} \chi_L$, and thereby generating M_u , are invariant under both Z'_2 and Z''_2 .

We supplement the symmetry Z_2 of (19) with

$$Z_2 : H' \rightarrow -H', \quad (22)$$

while H_R , H , and \tilde{H} transform trivially under Z_2 . Denoting the Yukawa-coupling matrices of H and H' by Y_d and Y'_d , respectively, the symmetry Z_2 leads to $(Y_d)_{22} = (Y_d)_{33}$, $(Y'_d)_{22} = -(Y'_d)_{33}$, and $(Y'_d)_{11} = 0$. The Yukawa-coupling matrix \tilde{Y} of \tilde{H} satisfies $(\tilde{Y}_d)_{22} = (\tilde{Y}_d)_{33}$. Then, with (A11) and (A12) of Appendix A, we obtain

$$M_d = \text{diag}(r + s, m + n + q, m - n + q), \quad (23)$$

$$M_\ell = \text{diag}(r - 3s, m + n - 3q, m - n - 3q), \quad (24)$$

where r , s , m , n , and q are complex parameters. This allows for the masses $m_d = |r + s|$ and $m_e = |r - 3s|$ to be unrelated. As for m_s , m_b , m_μ , and m_τ , they are given by only three effective parameters: n , $m + q$, and $m - 3q$. As $m_s \ll m_b$ and $m_\mu \ll m_\tau$, we find $m + q \simeq -n \simeq m - 3q$, and this in turn leads to the approximate relation [19]

$$m_\tau/m_b \simeq 1, \quad (25)$$

which is valid at the GUT scale. Clearly, m_s and m_μ remain unrelated. It is well known that (25) leads to the correct ratio m_τ/m_b at low energies [27, 28]. Thus, in our scheme one is able to accommodate all the down-type-quark and charged-lepton masses, and one is still rewarded with the correct relation (25) at the GUT scale.

2.5 The up-type-quark masses and the CKM angles

It remains to demonstrate that the known up-type-quark masses and CKM matrix can be accommodated through M_u . The scalar multiplets not coupling to ψ_R and ν_R will be denoted H_z (which are 5-plets) and \tilde{H}_z (45-plets). They are invariant under both Z'_2 and Z''_2 . First we consider the constraints from the family number F . The terms $(\chi_{L_a}^{ij})^T C^{-1} \chi_{L_b}^{kl}$ have F quantum numbers given by the matrix

$$\begin{pmatrix} 0 & +1 & -1 \\ +1 & +2 & 0 \\ -1 & 0 & -2 \end{pmatrix}. \quad (26)$$

Clearly, in order for the CKM matrix to be non-trivial we must allow for H_z and \tilde{H}_z to carry a non-zero family number $F = z$; this means that the subscript z gives, by definition, the F -value of the scalar multiplet. For the 5-plets we have the possibilities H_0 , $H_{\pm 1}$, and $H_{\pm 2}$; whereas for the 45-plets \tilde{H}_z , which couple through antisymmetric matrices, only $z = 0$ and $z = \pm 1$ have an impact on M_u . If, for a given pair of family indices (a, b) corresponding to a family number $F = -z$, there is only H_z , then we shall end up with $(M_u)_{ab} = (M_u)_{ba}$; if, on the contrary, there is no H_z but only \tilde{H}_z , then we shall have $(M_u)_{ab} = -(M_u)_{ba}$; if both H_z and \tilde{H}_z are present, then the matrix elements $(M_u)_{ab}$ and $(M_u)_{ba}$ will be unrelated; if neither H_z nor \tilde{H}_z exists, then $(M_u)_{ab} = (M_u)_{ba} = 0$.

Let us now proceed to take into account the symmetry Z_2 . We consider the above scalar multiplets $H_{0,\pm 1,\pm 2}$ and $\tilde{H}_{0,\pm 1}$. Under Z_2 we require that

$$Z_2 : \begin{cases} H_0 \rightarrow H_0, & H_1 \leftrightarrow H_{-1}, & H_2 \leftrightarrow H_{-2}, \\ \tilde{H}_0 \rightarrow -\tilde{H}_0, & \tilde{H}_1 \leftrightarrow \tilde{H}_{-1}. \end{cases} \quad (27)$$

We then find the following Yukawa couplings of the scalar 5-plets, compatible with Z_2 and with F :

$$\begin{aligned} \mathcal{L}_Y^{5,u} = & \epsilon_{ijklp} \left\{ (H_0)^p \left[a \left(\chi_{L1}^{ij} \right)^T C^{-1} \chi_{L1}^{kl} \right. \right. \\ & + b \left(\chi_{L2}^{ij} \right)^T C^{-1} \chi_{L3}^{kl} + b \left(\chi_{L3}^{ij} \right)^T C^{-1} \chi_{L2}^{kl} \left. \right] \\ & + c \left(\chi_{L1}^{ij} \right)^T C^{-1} \left[\chi_{L2}^{kl} (H_{-1})^p + \chi_{L3}^{kl} (H_1)^p \right] \\ & + c \left[\left(\chi_{L2}^{ij} \right)^T (H_{-1})^p + \left(\chi_{L3}^{ij} \right)^T (H_1)^p \right] C^{-1} \left(\chi_{L1}^{kl} \right) \\ & + d \left[\left(\chi_{L2}^{ij} \right)^T C^{-1} \chi_{L2}^{kl} (H_{-2})^p + \left(\chi_{L3}^{ij} \right)^T C^{-1} \chi_{L3}^{kl} (H_2)^p \right] \left. \right\} \\ & + \text{H.c.} \quad (28) \end{aligned}$$

With the 45-plets $\tilde{H}_{0,\pm 1}$ there are the following Yukawa couplings:

$$\begin{aligned} \mathcal{L}_Y^{45,u} = & \epsilon_{ijklp} \left\{ r \left(\tilde{H}_0 \right)_q^{lp} \left[\left(\chi_{L2}^{ij} \right)^T C^{-1} \chi_{L3}^{kq} \right. \right. \\ & - \left. \left. \left(\chi_{L3}^{ij} \right)^T C^{-1} \chi_{L2}^{kq} \right] \right. \\ & + t \left(\chi_{L1}^{ij} \right)^T C^{-1} \left[\chi_{L2}^{kq} \left(\tilde{H}_{-1} \right)_q^{lp} + \chi_{L3}^{kq} \left(\tilde{H}_1 \right)_q^{lp} \right] \\ & - t \left[\left(\chi_{L2}^{ij} \right)^T \left(\tilde{H}_{-1} \right)_q^{lp} + \left(\chi_{L3}^{ij} \right)^T \left(\tilde{H}_1 \right)_q^{lp} \right] C^{-1} \chi_{L1}^{kq} \left. \right\} \\ & + \text{H.c.} \quad (29) \end{aligned}$$

After the spontaneous breaking of Z_2 , the couplings in (28) yield a symmetric M_u ; if we have both (28) and (29), then we end up with a completely general up-type-quark mass matrix.

There is a lot of freedom in choosing among the possible scalar multiplets which may contribute to M_u , and one might think of deriving relations between the CKM mixing angles and the up-type-quark mass ratios. It is, however, difficult to imagine any such relation which might turn out to be in agreement with the known values of those quantities, since the up-type-quark mass ratios are unfavorably small. In the next section we shall simply assume a symmetric M_u generated by the five scalar 5-plets H_0 , $H_{\pm 1}$, and $H_{\pm 2}$, thereby discarding any possible 45-plets.

The family number F is softly broken, through terms of dimension 3, by the mass Lagrangian of the right-handed neutrino singlets; consequently, soft F -breaking terms must be considered also in the Higgs potential. Below the $SU(5)$ scale, F is effectively conserved in the Yukawa couplings of the leptons; at low scales its role is overtaken by the lepton numbers L_α . Thus, the idea of softly broken lepton numbers, advocated in [18], is compatible with an $SU(5)$ GUT. The family number F is also

spontaneously broken, at the weak scale, by the VEVs of the scalars with $F \neq 0$, which are needed for reproducing the known up-type-quark masses and CKM angles.

3 Gauge-coupling unification

Since we are constructing an $SU(5)$ GUT, we have to address the issue of gauge-coupling unification and we must check that things can be arranged in such a way that the unification scale m_U lies in the range 10^{16} to 10^{19} GeV; the lower value is determined by the need to avoid proton decay, the higher value corresponds to the Planck mass. We follow the strategy of [29–31] and use the one-loop renormalization-group equations (RGE) for the gauge couplings, which are decoupled from the RGE for the Yukawa couplings and for the scalar-potential couplings – see, e.g., [32]. We assume the “desert” hypothesis, i.e. that there are no particles with masses in between the Fermi scale (which we represent by the mass m_Z of the Z boson) and m_U . We then have

$$\frac{1}{\alpha_U} = \omega_1 - \frac{t}{2\pi} \left(\frac{41}{10} + a_1 \right) \quad (30)$$

$$= \omega_2 - \frac{t}{2\pi} \left(-\frac{19}{6} + a_2 \right) \quad (31)$$

$$= \omega_3 - \frac{t}{2\pi} (-7 + a_3). \quad (32)$$

In these equations, α_U is the fine-structure constant corresponding to the $SU(5)$ gauge coupling at the scale m_U , $t = \ln(m_U/m_Z)$, and $\omega_j = 1/\alpha_j(m_Z)$ for $j = 1, 2, 3$. The numbers $41/10$, $-19/6$, and -7 in (30)–(32) are the contributions to the RGE from the SM multiplets [32]; in particular, the numbers $41/10$ in (30) and $-19/6$ in (31) include the effects of the single Higgs doublet of the SM. The numbers a_1 , a_2 , and a_3 are the contributions to the RGE from any multiplets, beyond the SM ones, which might exist at (or below) the Fermi scale. Using ω_1 and ω_2 , which are rather well known, as inputs, while m_U and $\alpha_3(m_Z)$ are treated as outputs, one derives from (30)–(32) that

$$\ln(m_U/m_Z) = \frac{30\pi(\omega_1 - \omega_2)}{109 + 15(a_1 - a_2)}, \quad (33)$$

$$\alpha_3(m_Z) = \frac{2[109 + 15(a_1 - a_2)]}{3\omega_2[111 + 10(a_1 - a_3)] - 5\omega_1[23 + 6(a_2 - a_3)]}. \quad (34)$$

Numerically, we use

$$\begin{aligned} \alpha_3(m_Z) &= 0.1200(28), \quad \hat{\alpha}(m_Z)^{-1} = 127.934(27), \\ \text{and} \quad \sin^2 \hat{\theta}_W(m_Z) &= 0.23113(15), \end{aligned} \quad (35)$$

from the article by Erler and Langacker in [33]. In (35), α is the fine-structure constant and $\hat{\theta}_W$ is the weak mixing angle; the hats indicate that the $\overline{\text{MS}}$ renormalization scheme has been used in obtaining those quantities. Then, at the energy scale m_U , the values of

$$\alpha_1 = \frac{5}{3} \frac{\alpha}{\cos^2 \theta_W}, \quad \alpha_2 = \frac{\alpha}{\sin^2 \theta_W}, \quad (36)$$

Table 1. Contributions to a_j ($j = 1, 2, 3$) of the G_{SM} multiplets discussed in the text

	a_1	a_2	a_3
$(1, 2)_{1/2}$	1/10	1/6	0
$(6, 2)_{-1/6}$	1/15	1	5/3
$(1, 2)_{3/2}$	9/10	1/6	0
$(3, 1)_{2/3}$	4/15	0	1/6

and α_3 become identical, cf. (30)–(32). When applying (33) and (34), we use as input the mean values of $\hat{\alpha}(m_Z)^{-1}$ and $\sin^2 \hat{\theta}_W(m_Z)$ in (35), together with (36), for the computation of ω_1 and ω_2 .

The scalar representations $\underline{5}$ and $\underline{45}^*$ of $SU(5)$ each contain one Higgs doublet $(1, 2)_{1/2}$ (in the notation $(a, b)_c$ the numbers a and b are the dimensions of the representations of the $SU(3)$ and $SU(2)$ subgroups of $SU(5)$, respectively, while c is the value of the weak hypercharge). The VEVs of those Higgs doublets are of the order of the electroweak scale, and therefore the masses of those Higgs doublets, too, are at the Fermi scale. Since every $\underline{5}$ or $\underline{45}$ of $SU(5)$ supplies one light Higgs doublet, our model has (at least) nine Higgs doublets: one each from H_R , H , H' , \tilde{H} , H_0 , $H_{\pm 1}$, and $H_{\pm 2}$. This makes eight low-mass Higgs doublets beyond the one in the SM. From Table 1 we may compute the corresponding contributions to the a_j ; one obtains $10(a_1 - a_3) = 6(a_2 - a_3) = -15(a_1 - a_2) = 8$. Using (33) and (34), this leads to $m_U \simeq 8 \times 10^{13}$ GeV and $\alpha_3(m_Z) = 0.143$. The latter value is not too far from what is required, cf. (35), but the GUT scale m_U is much too low.

As a consequence of the preceding paragraph, we need some additional multiplets of G_{SM} at the electroweak scale, in particular some multiplets with non-trivial colour which might shift m_U to higher values while keeping $\alpha_3(m_Z)$ in the correct range. Let us denote such a candidate G_{SM} multiplet by D and investigate the conditions that we should impose on D . To avoid problems with proton decay, we require that D be embedded in an $SU(5)$ irrep which *cannot* have any Yukawa couplings; the lowest-dimensional eligible $SU(5)$ irreps are the $\underline{35}$ and the $\underline{40}$, see Appendix B. Moreover, since the scalars which have Yukawa couplings are 5 and 45-plets, D should not be contained in the decompositions of the $\underline{5}$, the $\underline{45}$, or their complex conjugates; else, D might, after the spontaneous breaking of the $SU(5)$ symmetry at m_U , mix with analogous G_{SM} multiplets from the $\underline{5}$ or $\underline{45}$, and thereby end up having proton-decay-generating Yukawa couplings. After imposing these two conditions, we find that there are indeed some satisfactory candidates: in particular, the $(6, 2)_{-1/6}$, which is contained in both the $\underline{35}$ and the $\underline{40}$ of $SU(5)$, and the $(1, 2)_{3/2}$, which is contained in the $\underline{40}$ (see Appendix B). The contributions of these multiplets of G_{SM} to the a_j are given in Table 1. In particular, we find that if, beyond the nine Higgs doublets, there are at the electroweak scale two $(6, 2)_{-1/6}$ and one $(1, 2)_{3/2}$, then

$$a_1 - a_2 = -5/3, \quad a_1 - a_3 = -3/2, \quad a_2 - a_3 = 1/6. \quad (37)$$

Using (33) and (34), we obtain the numerical result

$$m_U = 2 \times 10^{16} \text{ GeV}, \quad \alpha_3(m_Z) = 0.117. \quad (38)$$

This demonstrates that we may achieve a sufficiently high GUT scale and, simultaneously, reproduce $\alpha_3(m_Z)$ rather well. We stress that the $(6, 2)_{-1/6}$ and $(1, 2)_{3/2}$ do not occur in the decomposition of any of the $SU(5)$ irreps possibly coupling to fermions – see (13)–(15) and Appendix B. Therefore, couplings of the $(6, 2)_{-1/6}$ and the $(1, 2)_{3/2}$ to the SM fermions can only be induced by loop effects after G_{SM} breaking, and it is justified to assume that any such couplings will be very small. We need two light $(6, 2)_{-1/6}$ and one light $(1, 2)_{3/2}$, which we may take, for instance, from one $\underline{35}$ together with one $\underline{40}$ of $SU(5)$. The other G_{SM} multiplets in the $\underline{35}$ and $\underline{40}$ will have to be heavy, with masses of order m_U . This certainly means a fine-tuning problem for our theory, analogous to the well-known doublet–triplet splitting of the scalar 5-plets.

It is well known that gauge-coupling unification in the MSSM is compatible with the input data in (35) [29, 30]. The numbers in (37), which determine m_U and $\alpha_3(m_Z)$, are remarkable because they are exactly the same as in the minimal supersymmetric standard model (MSSM) [34]². The SM with eight extra Higgs doublets, two $(6, 2)_{-1/6}$, and one $(1, 2)_{3/2}$, produces exactly the same m_U and $\alpha_3(m_Z)$ as the MSSM, if we confine ourselves to the one-loop RGE. Differences will arise only at the two-loop level. Thus, the gauge-coupling unification of the MSSM can be imitated by simply adding a few scalar multiplets to the SM.

The choice of the G_{SM} multiplets $(6, 2)_{-1/6}$ and $(1, 2)_{3/2}$ displays one additional noteworthy feature. Let us assume that the total number of Higgs doublets is nine (including the SM doublet) and that the number of multiplets $(1, 2)_{3/2}$ is one, but let us allow the number n_6 of multiplets $(6, 2)_{-1/6}$ to vary. It turns out that in this case $\alpha_3(m_Z)$ is independent of n_6 and is always given by

$$\alpha_3(m_Z) = \frac{7}{12\omega_2 - 5\omega_1}. \quad (39)$$

Thus, numerically, the value given in (38) is precisely obtained from this formula, which is also valid for the one-loop RGE result of the MSSM. On the other hand, m_U does depend on n_6 , which can be chosen such that m_U lies in the correct range without putting at peril the good (39). The best choice is $n_6 = 2$, with the value of m_U given in (38). For $n_6 = 1$ the GUT scale comes down to 1.6×10^{15} GeV, which would result in much too fast proton decay, whereas for $n_6 = 3$ it increases to 1.2×10^{21} GeV, above the Planck mass.

We may replace the $(1, 2)_{3/2}$ by a $(3, 1)_{2/3}$; for the contributions to the a_j see again Table 1. Then, analogous to (38), we obtain

$$m_U = 4.1 \times 10^{17} \text{ GeV}, \quad \alpha_3(m_Z) = 0.117. \quad (40)$$

m_U is now higher than before, while $\alpha_3(m_Z)$ remains the same – by sheer coincidence, its value is again given by

(39). We note that with two $(6, 2)_{-1/6}$ and two $(3, 1)_{2/3}$ instead of one, we can even allow for eleven light Higgs doublets, with the result $m_U = 1.7 \times 10^{17}$ GeV and $\alpha_3(m_Z) = 0.123$. Using the $(3, 1)_{2/3}$ instead of the $(1, 2)_{3/2}$ makes a difference, though. The $(3, 1)_{2/3}$ does not only occur in the G_{SM} decomposition of the $\underline{40}$, it also occurs in the $\underline{10}^*$. This means that, after $SU(5)$ breaking, a coupling of the light scalar multiplet $(3, 1)_{2/3}$ to the fermionic bilinear $d_{\text{R}}^T C^{-1} d_{\text{R}}$ becomes allowed. Such a coupling will be induced at loop level. However, it depends on the coupling strength of the $\underline{40}$ to the 5 and 45-plets in the Higgs potential and it may in principle be made sufficiently small.

As for our simple usage of the one-loop RGE, refinements are, of course, possible. In particular, we might use the two-loop RGE, thereby taking into account the effect of the large Yukawa coupling of the top quark. It would also be possible to allow the light scalar multiplets to be somewhat heavier than m_Z , for instance with masses of order 0.5 or 1 TeV, like in the MSSM. Still another possibility would be to take into account various threshold effects at the scale m_U . Still, the short study above shows that there certainly are acceptable ways of making the gauge coupling constants unify in our $SU(5)$ theory with nine light Higgs doublets.

4 Conclusions

In [18] a simple extension of the lepton sector of the SM was put forward, with three right-handed neutrino singlets and the seesaw mechanism, and three Higgs doublets instead of one. By requiring conservation of the three lepton numbers in the Yukawa sector, while allowing them to be broken softly by the Majorana mass terms of the right-handed singlets, it was possible to enforce maximal atmospheric neutrino mixing by means of a Z_2 symmetry, while having arbitrary but in general large solar neutrino mixing. Since in this model maximal atmospheric neutrino mixing is enforced by means of a symmetry, the value 45° for the mixing angle is stable under radiative corrections.

In the present paper we have shown that the suggestion of [18] can be embedded in $SU(5)$ GUTs. Here we summarize the main features of the embedding.

- (1) Lepton mixing stems exclusively from the mass matrix M_{R} of the right-handed singlets ν_{R} ; atmospheric mixing is maximal; the solar mixing angle is free in general – without fine-tuning it will be large but not maximal; $U_{e3} = 0$. These are precisely the features of the tree-level mass matrix found in the model of [18] which, as we have now demonstrated, can be transferred to $SU(5)$ GUTs.
- (2) The CKM matrix is generated in the up-type-quark sector, while the down-type-quark mass matrix is diagonal. This is a consequence of the multiplet structure of $SU(5)$, in particular, of the 5-plet ψ_{R} in (12).
- (3) The family-number symmetry $U(1)_F$, which is responsible for the diagonal character of the matrices M_{D} in (17), M_d in (23), and M_ℓ in (24), is broken in two ways: *soft* breaking by M_{R} and by terms of dimension two and three in the Higgs potential, and *spontaneous* breaking by

² The individual a_j are different, however

the VEVs of the scalar multiplets responsible for the up-type-quark mass matrix. The non-trivial CKM matrix is obtained via the spontaneous breaking of $U(1)_F$, and the non-trivial lepton-mixing matrix is obtained via the soft breaking of $U(1)_F$.

(4) On the other hand, Z_2 , which is responsible for maximal atmospheric neutrino mixing once the charged-lepton mass matrix is diagonal, is broken only by the VEVs.

(5) In the lepton sector, the $U(1)_F$ enforces diagonal Yukawa couplings. Therefore, in this sector and below the GUT scale, instead of the family number F we have the usual three lepton numbers L_α ($\alpha = e, \mu, \tau$), which are only *softly* broken by the mass terms of the right-handed singlets [18, 24].

These features are probably the most generic ones of our model. The extension of the model of [18] to an $SU(5)$ GUT is certainly not unique, and the discussion in this paper should be perceived as just an existence proof for that extension. Among the features which might depend on the way the extension is performed, we may count the following ones.

(1) The assignment of family numbers chosen by us, together with our Z_2 symmetry, can be thought of as originating from a non-abelian $O(2)$ symmetry group.

(2) Our choice of scalars coupling to the down-type-quark sector produces the successful relation $m_b \simeq m_\tau$ at the GUT scale.

(3) We need eight scalar 5-plets and one 45-plet in order to accommodate maximal atmospheric neutrino mixing, the charged-fermion masses, and CKM mixing. It is, therefore, natural to assume that in our model there are nine light Higgs doublets. We have shown that it is nevertheless possible to obtain gauge-coupling unification, even when assuming a desert between the electroweak and GUT scales.

We want to stress that obtaining gauge-coupling unification in our model is not trivial at all. However, we were able to find an excellent solution, in which the light scalar multiplets are nine Higgs doublets $(1, 2)_{1/2}$, one doublet $(1, 2)_{3/2}$, and two $(6, 2)_{-1/6}$. It is most remarkable that in this case the one-loop RGE for the gauge couplings lead to results identical to those in the MSSM. Finally, we remark that in our embedding the up-type-quark mass matrix is a general symmetric mass matrix, with no relationships among the up-type-quark masses and the CKM angles; it is possible that more predictive embeddings exist.

Acknowledgements. W.G. thanks H. Stremnitzer for many helpful discussions on $SU(5)$.

Appendix

A Yukawa couplings

In the following, indices which transform through the matrix $U \in SU(5)$ are written as upper indices; indices transforming through the matrix U^* are written as lower indices [20]. It is clear from (13), (14), and (15) that the scalars which may have Yukawa couplings to ψ_R and/or χ_L must

be in one of the following representations of $SU(5)$: $\underline{5}$, $\underline{10}$, $\underline{15}$, $\underline{45}$, or $\underline{50}$. We want to avoid spontaneous violation of colour or electric charge, and to disallow a Majorana mass term for the left-handed neutrinos. The scalar multiplets present in Yukawa couplings are, therefore [20], $H^i \sim \underline{5}$ and $\tilde{H}_k^{ij} \sim \underline{45}^*$; the latter satisfies $\tilde{H}_k^{ij} = -\tilde{H}_k^{ji}$ and $\tilde{H}_i^{ij} = 0$ (we use the summation convention). The Yukawa couplings are given by [20]

$$\mathcal{L}_{\text{Yukawa}} = \mathcal{L}_Y^5 + \mathcal{L}_Y^{45}, \quad (\text{A1})$$

with

$$\mathcal{L}_Y^5 = \bar{\psi}_{Ria} \chi_{Lb}^{ij} H_j^* (Y_d)_{ab} \quad (\text{A2})$$

$$- \frac{1}{8} \epsilon_{ijklp} \left(\chi_{La}^{ij} \right)^T C^{-1} \chi_{Lb}^{kl} H^p (Y_u)_{ab} + \text{H.c.},$$

$$\mathcal{L}_Y^{45} = \frac{1}{2} \bar{\psi}_{Ria} \chi_{Lb}^{jk} \tilde{H}_{jk}^{*i} (\tilde{Y}_d)_{ab} \quad (\text{A3})$$

$$- \frac{1}{8} \epsilon_{ijklp} \left(\chi_{La}^{ij} \right)^T C^{-1} \chi_{Lb}^{kq} \tilde{H}_q^{lp} (\tilde{Y}_u)_{ab} + \text{H.c.}$$

The numerical factors in these equations are conventional. The symbol ϵ_{ijklp} represents the completely antisymmetric tensor, which is normalized through $\epsilon_{12345} = +1$. The indices a and b are flavor indices. The Yukawa-coupling matrices Y_d and \tilde{Y}_d are general complex 3×3 matrices; the matrix Y_u is symmetric without loss of generality, while \tilde{Y}_u is antisymmetric:

$$(Y_u)_{ab} = (Y_u)_{ba} \quad \text{and} \quad (\tilde{Y}_u)_{ab} = -(\tilde{Y}_u)_{ba}. \quad (\text{A4})$$

The vacuum expectation values are given by [20]

$$\langle H^i \rangle_0 = \frac{v}{\sqrt{2}} \delta_5^i \quad (\text{A5})$$

and

$$\langle \tilde{H}_k^{i5} \rangle_0 = -\langle \tilde{H}_k^{5i} \rangle_0 = \frac{\tilde{v}}{\sqrt{2}} (\delta_k^i - 4\delta_4^i \delta_k^4) \quad \text{for } i \leq 4,$$

$$\langle \tilde{H}_k^{ij} \rangle_0 = 0 \quad \text{for } i, j \leq 4. \quad (\text{A6})$$

The charged-fermion mass matrices are defined by

$$\mathcal{L}_{\text{mass}} = -\bar{u}_R M_u u_L - \bar{d}_R M_d d_L - \bar{\ell}_R M_\ell \ell_L + \text{H.c.} \quad (\text{A7})$$

The relations

$$\epsilon_{ijk45} \left(\chi_{La}^{ij} \right)^T C^{-1} \chi_{Lb}^{k4} = 2 \bar{u}_{Ra} u_{Lb}, \quad (\text{A8})$$

$$\epsilon_{ijkl5} \left(\chi_{La}^{ij} \right)^T C^{-1} \chi_{Lb}^{kl} = 4 (\bar{u}_{Ra} u_{Lb} + \bar{u}_{Rb} u_{La}), \quad (\text{A9})$$

which follow directly from the components of χ_L given in (12), are useful for the extraction of the up-type-quark mass matrix M_u . One obtains

$$M_u = \frac{1}{\sqrt{2}} \left(v Y_u - 2\tilde{v} \tilde{Y}_u \right), \quad (\text{A10})$$

$$M_d = \frac{1}{\sqrt{2}} \left(v^* Y_d + \tilde{v}^* \tilde{Y}_d \right), \quad (\text{A11})$$

$$M_\ell = \frac{1}{\sqrt{2}} \left(v^* Y_d^T - 3\tilde{v}^* \tilde{Y}_d^T \right). \quad (\text{A12})$$

B Branching rules

In this appendix we display the branching rules for some representations of $SU(5)$ in terms of representations of G_{SM} . For simplicity we do not underline the dimensions of the representations of $SU(3)$ and $SU(2)$. The weak hypercharge Y is normalized in the usual SM way, i.e. $Y = Q - T_3$. The branching rules below may, for instance, be found in [25]³.

The defining representation of $SU(5)$ is

$$\underline{5} = (3, 1)_{-1/3} + (1, 2)_{1/2}. \quad (\text{B1})$$

The product of two $\underline{5}$'s yields

$$\underline{15} = (6, 1)_{-2/3} + (3, 2)_{1/6} + (1, 3)_1, \quad (\text{B2})$$

$$\underline{10} = (3^*, 1)_{-2/3} + (3, 2)_{1/6} + (1, 1)_1. \quad (\text{B3})$$

The representations $\underline{45}$ and $\underline{50}$ of $SU(5)$, which arise in the tensor products of fermionic representations in (14) and (15), have the following branching rules:

$$\begin{aligned} \underline{45} = & (3, 1)_{-4/3} + (8, 2)_{-1/2} + (1, 2)_{-1/2} + (6, 1)_{1/3} \\ & + (3^*, 3)_{1/3} + (3^*, 1)_{1/3} + (3, 2)_{7/6}, \end{aligned} \quad (\text{B4})$$

$$\begin{aligned} \underline{50} = & (6^*, 1)_{-4/3} + (8, 2)_{-1/2} + (6, 3)_{1/3} + (3^*, 1)_{1/3} \\ & + (3, 2)_{7/6} + (1, 1)_2. \end{aligned} \quad (\text{B5})$$

Finally, in Sect. 3 we use the irreps $\underline{35}$ and $\underline{40}$ and their decompositions:

$$\begin{aligned} \underline{35} = & (10, 1)_{-1} + (6, 2)_{-1/6} + (3, 3)_{2/3} \\ & + (1, 4)_{3/2}, \end{aligned} \quad (\text{B6})$$

$$\begin{aligned} \underline{40} = & (8, 1)_{-1} + (6, 2)_{-1/6} + (3^*, 2)_{-1/6} + (3, 3)_{2/3} \\ & + (3, 1)_{2/3} + (1, 2)_{3/2}. \end{aligned} \quad (\text{B7})$$

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³ Note, however, that the irreps $\underline{35}$, $\underline{40}$, $\underline{45}$, and $\underline{50}$ used here correspond to their respective complex conjugates in [25]; in the definitions of $\underline{45}$ and $\underline{50}$ we have followed [20]

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